

CS 2461

Lab- Week 5

# Today....

- Review Design of Finite State Machines – example
- Bitwise operations in C – complete the exercise from lecture course webpage – Exercises-Week5 download sept27.c

# Finite State Machines

- The behavior of sequential circuits can be expressed using finite state machines (FSMs).
  - FSMs consist of a set of nodes that hold the states of the machine and a set of arcs that connect the states.
  - FSM represented as a graph
- Elements of FSM:
  - Finite Number of states
  - Finite number of inputs and Finite number of outputs
  - A specification of the state transitions
  - A specification (Boolean function) of the outputs
  - Outputs are associated with a node/state

# FSM Design Process

- The first step is to model the behavior of the machine
  - Based on problem statement
  - Identify what the inputs are
  - Identify the outputs
  - Determine what needs to be stored to capture the “state” of the machine
- Represented as a graph – finite state diagram
  - Nodes: States – a state stores summary of events (until current time)
  - Edges: Transition from current state to next state
    - Based on input and current state and computed by combinational logic
  - Outputs: value of outputs at each state
- State: the **state** of a system is a “**snapshot**” of all relevant elements at an instant in time.
  - Example : vending machine should remember total money received,....

# Designing and implementing a FSM

1. Understand the problem statement and determine inputs/outputs
2. Identify states and draw the state diagram
  - Encode each state in binary using N bits
  - State diagram will show transitions from state to state based on value of inputs
3. Next, derive the truth table (from state diagram)
  - “inputs” in truth table are N current state variables and the inputs
  - These N state variables will need to be stored in N flips flops,
  - Label the N state variables  $S_{N-1} S_{N-2} \dots S_1 S_0$
  - “outputs” are the values of the state variables in the next state and the output at each state -- common notation is  $S'$  but confusion with complement operator, so let's use  $S^*$
4. From truth table, implement combinational circuit (boolean function) for each of the next state values & outputs
  - State variables are stored in your N storage elements

# FSM Design: Example

- design the finite state diagram for a sequential circuit that generates an output  $Z=1$  whenever the input (binary) string it has read thus far has an odd number of 0's and an odd number of 1's.
  - For example, if the input string is 010010 (4 zeros and 2 ones) then output  $Z=0$ . If the input string is 1101 ( 1 zeros and 3 ones) then the output is  $Z=1$ .
- Assume: at each clock cycle the machine reads one bit (a 0 or 1)
  - Eg. If overall input = 0101 then after 2 cycles it would have read 01 (and output=1),  
after 3 cycles it has read 010 (and output =0), etc.

## Question: What property in a state ?

- The first 'property' to note is that for any binary string, the string has some  $X$  number of 1's and some  $Y$  number of 0's.
  - For string 01100 we have  $X=2$  and  $Y=3$
- The question is now defined in terms of the properties of  $X$  and  $Y$ 
  - if  $X$  and  $Y$  are both odd then output  $Z=1$
- For any binary string how many cases can you have in terms of the evenness/oddness of  $X$  and  $Y$  ?
- Four cases = Four states:
  1.  $X$  even and  $Y$  even
  2.  $X$  even and  $Y$  odd
  3.  $X$  odd and  $Y$  even
  4.  $X$  odd and  $Y$  odd

## State transitions

- you have 4 cases for any binary string of any length; next, what happens if a string that has been processed (i.e., read) thus far falls under case 3 (i.e.,  $X$  is odd and  $Y$  is even) and then the machine reads a 1
- = String (thus far) now has  $X$  is even and  $Y$  is even = Case 1
- you have 4 cases for any binary string of any length; next, what happens if a string that has been processed (i.e., read) thus far falls under case 3 (i.e.,  $X$  is odd and  $Y$  is even) and then the machine reads a 0
- =  $X$  is odd and  $Y$  is odd = Case 4



# Complete the transitions

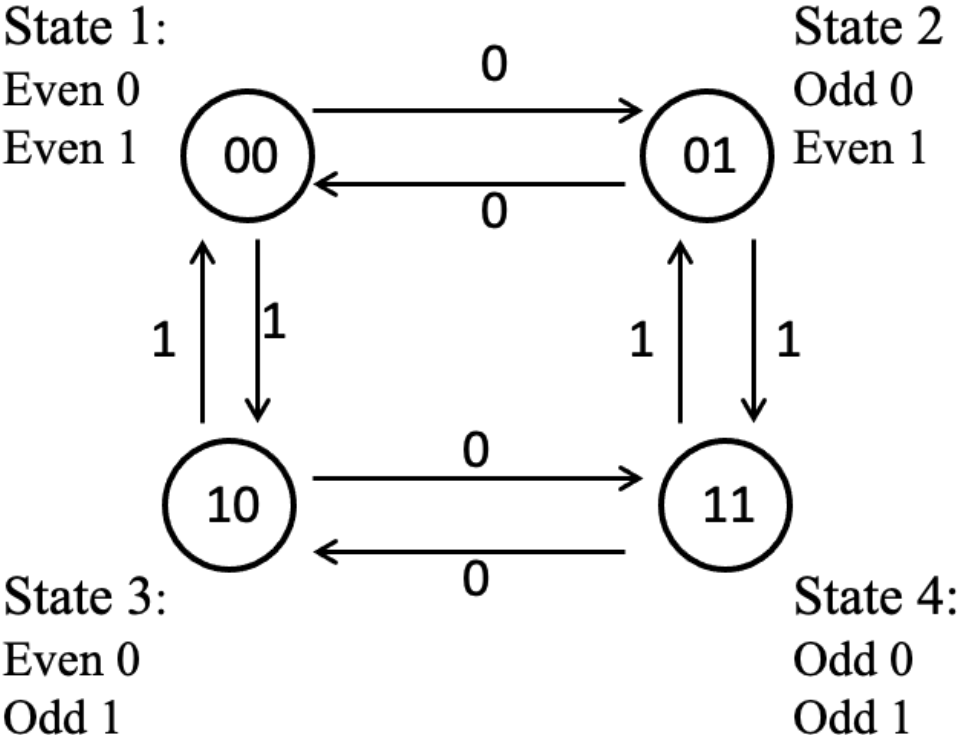
- Case1 (State1):  $X$  even and  $Y$  even
  - Read 0 go to ?
  - Read 1 go to ?
- Case 2 (State 2):  $X$  even and  $Y$  odd
  - Read 0 go to ?
  - Read 1 go to ?
- Case 3 (State 3):  $X$  odd and  $Y$  even
  - Read 0 go to ?
  - Read 1 go to ?
- Case 4 (State 4):  $X$  odd and  $Y$  odd
  - Read 0 go to ?
  - Read 1 go to ?

$X$  = no. of 1's in string  
 $Y$  = no. of 0's in string

*States:*

1.  $X$  even and  $Y$  even
2.  $X$  even and  $Y$  odd
3.  $X$  odd and  $Y$  even
4.  $X$  odd and  $Y$  odd

# Draw the finite state diagram



# Truth table

- How many state variables (storage bits) ?
- 2 bits –  $S_1 S_0$ 
  - State1 = 00      State2 = 01    State3 = 10    State4 = 11
- If FSM is in State1 (00) and input=0 then next state is 01 and current output in State00 is  $Z=0$
- $In=0$   $S_1=0$   $S_0=0$  then  $S_1^*=0$   $S_0^*=1$   $Z=0$
- If FSM is in State 4 (11) and input=1 then next state is 01 and current output in State11 is  $Z=1$
- $In=1$   $S_1=1$   $S_0=1$  then  $S_1^*=0$   $S_0^*=1$   $Z=1$

# Complete the Truth Table

In	S1	S0	S1*	S0*	Z
0	0	0	0	1	0
0	0	1	0	0	0
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	0
1	1	1	0	1	1

# Questions on Finite State Machines ?

# Bitwise operations in C

Go to course website

Download Exercises Week 5 – Sept27.c (under C and Data Rep)  
(do not compile and run it yet!)

# C data types and operators

- Unsigned and signed int – C allows casting
- Bitwise operations:
  - & (and)
  - | (or)
  - ~ (complement)
  - ^ (XOR)
  - Right shift >>     arithmetic shift = MSB is replicated rightwards shift
  - Left shift <<     shift in 0's into the LSBs
    - What is left shift one position ? What is the value of  $(x \ll 1)$  ?
- Logical operators: arguments are treated as binary (True or false)
  - && (logical And)
  - || (logical OR)
  - ! (logical NOT)
  - Key takeaway: if x is a non-zero integer then x is True

# Time to test your C ...

- Download/open Exercises-Week5 (a C file called sept27.c ) from webpage
- **Do NOT run the C code**....Go through code and answer the questions without running the code
  - Reading code (without running it) is a very important skill
- Use the following values as inputs (this info included in the comments in C file):
  - `zz = abcd0123` (hex representation of a 32 bit number)
  - `a=4, b=7, n=2` and different values of `c` for `c>0, c<0, and c=0`
- 1. First answer the questions
- 2. Next: Compile and run your code & compare your answers with the run-time results
- 3. Can you explain what is going on (if your answer did not match)

Notation: a prefix of `0x` indicates Hex representation

`0x25` is the integer  $2*16^1 + 5*16^0 = 37$

`0xFF` is the integer with 1's in last 8 bits, i.e., decimal value 255



# Describing the function

- CallMeNext

```
int CallMeNext(int x){  
    int t;  
    t= (1 <<x);    what is 1 << 1 ? What is 1 << 2 ?  
    t= t+1;        what is 1 << x ?  
    return (t);  
}
```

# Describing the function

- CallMeLast

```
int CallMeLast(int x, int y){
    int temp;
    temp = ~y;    computes complement of y (invert all bits)
    temp = temp +1;    what is temp= (NOT y)+1 ?
    temp = temp + x;
    return(temp);
}
```

# Describing the function

- whoaml  $0 \leq n \leq 3$

```
int whoamI(int x, int n){
    int rs;
    int y = n << 3;  what is y when n=2 ? What happens when you
                     shift an integer left 3 places ?
    int xs = 0xFF << y;  what is 0xFF left shifted 16 places
    rs=  xs & x;        what bits (bytes) are you masking?
    /* return(rs); */
    return ((rs >> y)& 0xFF);
}
```

Example:  $x = 0x\text{abcd1234}$  and  $n=2$

What is  $y = 2 \ll 3 = 2 * 8 = 16$

What is  $xs = 0xFF \ll 16 = 0x\text{00FF0000}$

What is  $rs = 00FF0000 \& \text{abcd1234} = ?$

# Describing the function

- WhatamI

```
int whatamI(int A) {  
    int X,Y;  
    X = (A ^ (-A)) >> 31; suppose Z= (A ^ (-A)) then what is the least  
        significant bit of X in terms of Z?  
    Y = (X & 0x1); when is Y=1 ? When is Y=0 ?  
    return(Y);  
}
```

think of the binary representation of integers A and -A

if A=12 in binary = 0000.....1010 and 0x 0000 000C in Hex

What is -A = -12 in binary ?

then what is the MSB of (A ^ (-A))